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EXTRACTION OF IMPACT OF WIND TURBULENCE ON RC HELICOPTERS USING MACHINE LEARNING

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ABSTRACT

This paper presents a new cost effective wireless telemetry system capable of estimating ambient air turbulence using RC helicopters. The proposed telemetry system correlates the RC helicopter's flight dynamics with ship air wake patterns generated by cruising naval vessels. The telemetry system consists of two instrumentation units each equipped with aviation grade INS/IMU sensors to measure dynamics of the helicopter with respect to the concerned naval vessel. The presented telemetry system extracts ship air wake patterns by removing the helicopter dynamic effects from actual measurements. This paper presents a comprehensive comparison between popular machine learning algorithms in eliminating effects of pilot inputs from helicopter's dynamics measurements. The system was tested on data collected in a wide range of wind conditions generated by modified YP676 naval training vessel in the Chesapeake Bay area over a period of more than a year.

NOMENCLATURE

INS	Inertial Navigation System
IMU	Inertial Measurement Unit
SAW	Ship Air Wake
PWM	Pulse Width Modulation (μs)
RC	Radio Controlled
BPNN	Back Propagation Neural Network
CFD	Computational Fluid Dynamics
ML	Machine Learning
VTOL	Vertical Take-Off and Landing
BME	Bayesian Mixture of Experts
ANFIS	Adaptive Neuro Fuzzy Inference System

I. INTRODUCTION

With a limited flight deck area, launch and recovery of VTOL aircrafts from naval warships is one of the most challenging tasks in naval aviation [1]. In addition to the 'rocking' motion of the ships operating in high seas, local air turbulence acts as an important factor to be considered for the safety of naval aircrafts. Ship air wakes are the trails of air turbulence left behind by the superstructure of naval ships. Ship air wakes create a high risk environment for operation of helicopters on landing decks. The interaction between the rotor wakes and ship air wake often results in unexpected motion of the helicopter. As the landing area on naval ships is limited, such unexpected motion pose potential hazard to both ship and helicopter.

To minimize the operational risks due to ship air wakes, safe launch and recovery envelops are prescribed and used while operating VTOLs (helicopters) from cruising naval ships. These safe operational envelops depend on ship structure and are estimated by manual fight testing before any ship gets clearance for operating helicopters. This aircraft/naval vessel rating procedure, is not only expansive but highly dangerous and risky as the pilots are asked to perform in an unknown air turbulence pattern with limited flight deck area for landing and takeoff. In addition to the risks involved, such procedures are highly subjective since each pilot perceives impact of ship air wakes differently. Use of Computational Fluid Dynamics (CFD) models is a potential solution to this problem, but existing models are not mature enough to estimate ship air wake accurately [2-7]. Such CFD models require rigorous validation which further require unbiased test data [8-10]. The proposed telemetry system is designed to acquire this experimental ship air wake data.

To obtain experimental ship air wake data, researchers either directly read wind velocities using anemometers arrays, or use scaled down wind tunnel measurements. Use of anemometers is easy but proves to be expansive and time consuming. In addition, the use of anemometers is limited to deck area only since they cannot be deployed for off board ship air wake measurements. Use of wind tunnel measurement for the estimation of ship air patterns has two main limitations: First, the turbulence patterns generated by scaled down models in wind tunnels are not linearly scaled. Second, both the model holder and wind tunnel walls affect the readings and would need to be accounted for. These factors motivate the need for a

wireless instrumentation system capable of measuring ship air wakes from an off-board platform.

Due to their relatively low mass, RC helicopters are more sensitive to ship air wakes compared to full-scale helicopters. This makes RC helicopters an ideal platform to measure ship air wakes. Use of RC helicopter's vibrations to measure ship air wakes was first explored by [11]. However, this work was limited since it lacked any compensation for helicopter's own vibrations due to pilot inputs. In addition, the system used onboard data loggers to record position and vibration data separately resulting in data that was not properly synchronized.

This paper extends previous research [10, 12-15] by incorporating real-time wireless communication and compensation for pilot input in helicopter dynamics. The presented work focuses mainly on software and system modelling details of the latest iteration of the system and improvements in pilot input compensation. The main benefit of this system is its untethered long-range mobility enabling wide area coverage for data collection that does not alter the air wake patterns. Furthermore, the pilot input compensation features of the system ensures the unbiased ship air wake measurements.

The previous work on this telemetry system [15] used only Back Propagation Neural Network (BPNN) to model the helicopter's response to pilot inputs. This work presents three different machine learning technique: 1) BPNN, 2) Bayesian Mixture of Experts (BME) and 3) Adaptive Neuro-Fuzzy Inference System (ANFIS). These three ML algorithms were selected for modelling the system since they represent the simplest of the three basic categories in ML algorithm. BPNNs use error minimization and represent algorithms using gradient descent methods for to fit complex non-linear models on data. BMEs use simpler models (linear experts) to model complex data pattern and probabilistically mix them using expectation maximization. Thus, BMEs represent a class of Bayesian learning algorithms. ANFIS networks are fuzzy inference systems which use gradient-descent-type algorithms to tune fuzzy membership functions. As a result, ASFIS networks possess capabilities of both BPNNs (as they use error minimization for parameters optimization) and BMEs (as they use simple models during defuzzification process). As these ML algorithms are the simplest of their kind, prediction results from these algorithms will be similar to other complex algorithms of these kinds.

II. TELEMETRY SYSTEM

Wind turbulence is stochastic spatial variance of wind velocity and is often modelled on the basis of wave number of local velocity pattern [16]. Wind turbulence results in uneven wind loading of the rotor blades and adds undesirable heaving and oscillations to helicopter motion. Wind perturbations can be decomposed into two components: one in the plane of rotation of the main rotor blade (called in-plane turbulence) and another normal to it (called out-of-plane turbulence). The in-plane wind turbulence can be of thought of as an uneven wind distribution about the rotor hub. During continuous rotation, the motion of the rotor blades is in same direction as the wind for of a rotation cycle and opposite for the other half. In addition to this rotation effect, the unequal planer wind velocities also results in differential lift generation. This uneven lift makes the helicopter tilt (or even rotate) if not controlled. Similarly, the out-of-plane turbulence differentially changes the rotor influx which further results in differential thrust and tilting. This unexpected tilting can be detected by a gyroscope in the form of angular rates. The proposed wireless telemetry system uses an aviation grade INS/IMU sensor capable of reading helicopter's fuselage angular rate and attitude at a high sampling rate.

The proposed system consists of two independent instrumentation sub-systems called rover module and base module [15]. These modules communicate with each other via a long range Wi-Fi network with an update rate of up to 150 Hz. The Wi-Fi router utilizes two types of antennas, one Omni-directional short-range rod antenna and another long range direction Yagi antenna. Fig. 1 shows hardware setup for the presented telemetry system.

To estimate ship air wake patterns, an RC helicopter retrofitted with the rover module is flown in the target areas.
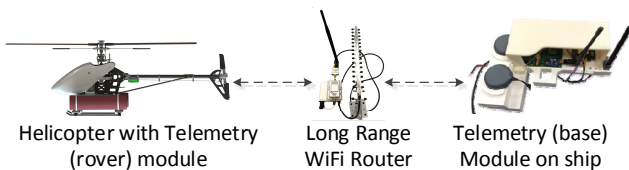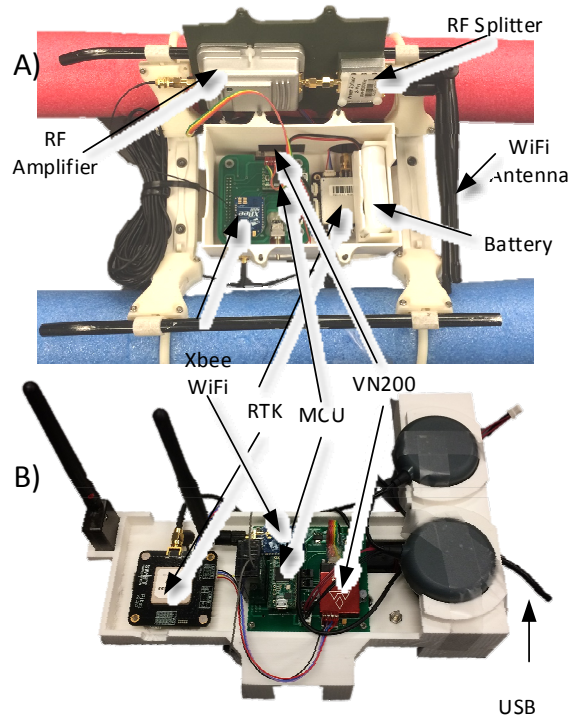

Figure 2. Telemetry System : A) Bottom view of Rover Module with cover removed; B) Rear view of base Module on plastic mount

Figure 1. Telemetry System hardware setup

The rover module then sends the helicopter's dynamics data to the base module over Wi-Fi. A computer connected to the base module records and processes the data and displays appropriate results/flight parameters on screen in the form of graphs and trajectories. Fig. 2 shows the annotated images of the rover module and the base module.

During measurement, the proposed telemetry system is mounted on T-REX 600E PRO RC helicopter and flown in the lee (downwind) of the superstructure of the YP676 craft in a sweeping trajectory. The data received on the base computer processes and records the data in real-time. The relative position of the helicopter in the boat's frame of reference is obtained from position and heading estimates from the VN200 INS on both modules. During post processing of the data, the system allows the user to select and use one of three different machine learning algorithms to process the data. The selected algorithm filters away pilot input components from angular rate measurements of helicopter and estimates the air wake impact on the helicopter. The air wake intensity in the form of dynamics of helicopter arising from air wakes is then plotted on helicopter trajectory relative to the ship. The YP676 is equipped with anemometer array to help the craft master to maintain constant relative wind conditions. Fig. 3 shows the telemetry system operating over the flight deck of the modified YP676 craft.
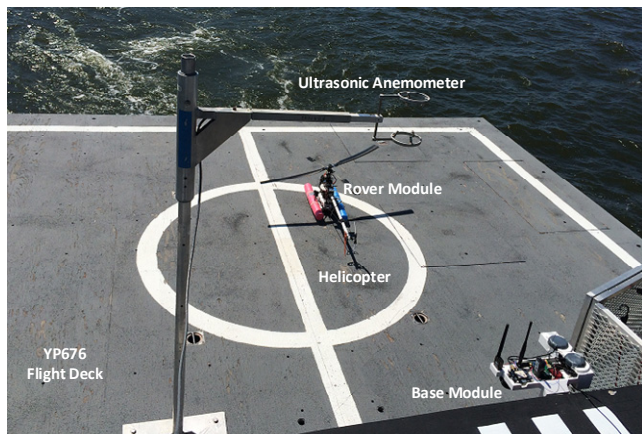

Figure 3. Telemetry system operating on YP746 flight

III. PILOT INPUT COMPENSATION

The presented telemetry system uses the helicopter's angular rates measured using a gyroscope to characterize ship air wakes. The biggest challenge in this process is that ship air wakes are not the only source of vibrations in helicopter. In addition to air wakes, vibrational noise due to the moving parts in the helicopter and pilot inputs/maneuvers constitutes a major component in helicopter vibration measurement. The vibrations due to ship air wakes can be extracted by removal of pilot induced vibrations and harmonic/vibrational noise from raw angular measurement.

Helicopter motion is primarily controlled with a swash plate mechanism through cyclic control input. During flight, other than the effects of air wakes/turbulence, the main forces acting on the helicopter are rotor blade thrust, gravity, gyroscopic forces and air drag [16]. Fig. 4 shows free body diagram of the helicopter where yellow arrows represent forces, green arrows velocities and blue arrows frame of reference. A forward motion (Fig. 4) experiences a differential thrust (F_1 and F_2) along the longitudinal axis is applied through cyclic control. This different thrust makes helicopter tilt forward [17].

Any tilting or vibration of the helicopter disturbs the rotating rotor blades and stabilization bar from their plane of rotation and generates a gyroscopic torque (T_{gy}) opposing the rotation of helicopter. While tilting forward, in addition to gyroscopic torque, helicopter experiences a pendulum like counter torque as the point of rotation (rotor hub) is higher than the center of mass of the helicopter. Due to high speed downwash from rotor, motion of helicopter faces significant viscous drag [15]. All these opposing torques make the angular velocity of the helicopter a non-linear function of pilot inputs and attitude. In this paper, the authors have modelled the helicopter's non-linear response to pilot inputs in using the machine learning techniques BPNN, BME and ANFIS networks.

IV. SYSTEM TRAINING

For training the system, the telemetry system was flown in a large enclosed hangar in Davison Air Field. The hanger provided an air wake free zone to collect the training data for the machine learning algorithms. Four indoor flights were conducted with T-REX 600E Pro helicopter to collect training data. For collection of a versatile data set, the pilot was instructed to fly the helicopter in a variety of extreme maneuvers. To avoid ground effect disturbances [18] in training data, the helicopter was flown at a significant height (at least 2 rotor diameters ~3m) from floor. During these training flights, a variety of helicopter maneuvers were performed. A versatile dataset of pilot inputs along with helicopter attitude with angular rates was created during these indoor experiments.
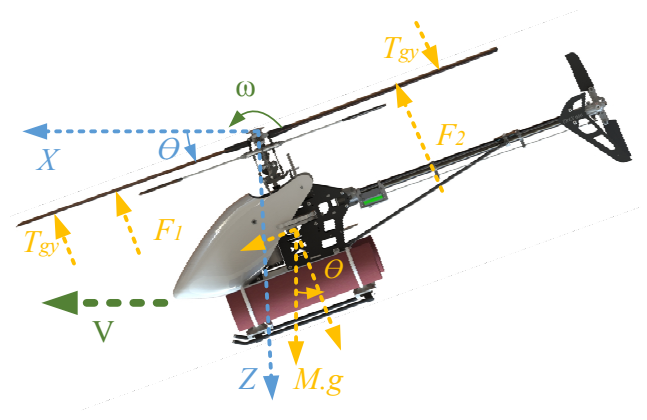

Figure 4. Free body diagram of helicopter

During these four indoor flights, a dataset of around 75000 samples was collected. To ensure the prediction generalization of the system, only 6.25% of the total data was used for training. The remaining data was used for testing the performance of the system.

For each of the three machine learning algorithms, three networks were trained to individually predict the three Cartesian components of helicopter's angular rates from pilot inputs and attitude information. Thus, each network was trained with a 6-dimensional input vector consisting of the 3 swash plate servo PWM signals, 1 tail rotor PWM signal and 2 attitude (pitch and roll) angles. The output vector was a one dimensional Cartesian component of helicopter's angular rates. To improve the prediction accuracy of the networks, the complete dataset of input and output vectors was normalized to zero mean and unit standard deviation before training.

Due to the cyclic nature of rotor input, the rotor thrust in a particular direction (plane passing through rotor hub) is pulsatory in nature. Since the other forces acting on helicopter are continuous in nature, these pulses of rotors show up the form of high frequency vibrational noise in the gyroscope reading and can be removed using frequency domain filtering. Since random sensor noise from the gyroscope is relatively small in comparison to this pulsatory high frequency noise, it has been neglected. In this paper, the authors have used an optimized low-pass Gaussian filter to remove this high frequency noise from data. To extract the signature of this pulsating noise, the helicopter was kept hovering for some time in each training flight. During this hovering period, minimum pilot inputs were provided so the helicopter so that only noise is captured by the system. Gaussian low pass filters with different parameters were applied on the hovering data until the noise was removed. During this experiment, a Gaussian low pass filter with a cutoff frequency of 11 Hz and kernel length of 1 s gave best results. The obtained filter cutoff frequency seems reasonable as the pilot input frequency never exceeded 5-6 Hz.

To extract the ship air wake pattern, the pilot input component (predicted from the machine learning network) was subtracted from low pass filtered recorded data. To avoid any adverse effect on training process, the low pass filter was not applied on data before training.

A. Training BPNN

BPNN is a multilayer feed-forward network and uses error back propagation algorithm for training [19-22]. For prediction for pilot input component in helicopter's angular rates, 3 BPNNs each with 2 hidden layers were trained. The number of nodes in input and output layers were selected as 6 and 1 based

Table 1. BPNN topologies

Neural Network	Input Layer	Hidden Layer 1	Hidden layer 2	Output layer
Net1 (X axis)	6	5	5	1
Net2 (Y axis)	6	5	7	1
Net3 (Z axis)	6	4	8	1

on the dimensionality of the input and output vectors. The number of neurons in hidden layers were selected by trial and error. The number of neurons in both hidden layers were varied from 3 to 15. As weights of the BPNNs are selected randomly and the network converges at local optima in weight space, each network topology was trained 15 times. The network topology delivering the best prediction accuracy was retained. MATLAB's 'Neural Network Toolbox' was used for error back propagation training with the Levenberg–Marquardt algorithm [23-24]. The "tansig" and "purelin" were used as activation functions for the hidden layers and the output layer respectively [25]. Ten-fold cross validation method [26] was used to prevent overtraining of the networks. Table 1 shows the final topologies for the three trained BPNNs.

B. Training BME

Bayesian Mixture of Experts (BME) is a type of directed graph network in which independent output of multiple subunits (Experts) are probabilistically combined into a single output [27]. Each expert consists of an observed continuous node and a hidden discrete node. The continuous hidden node (or output
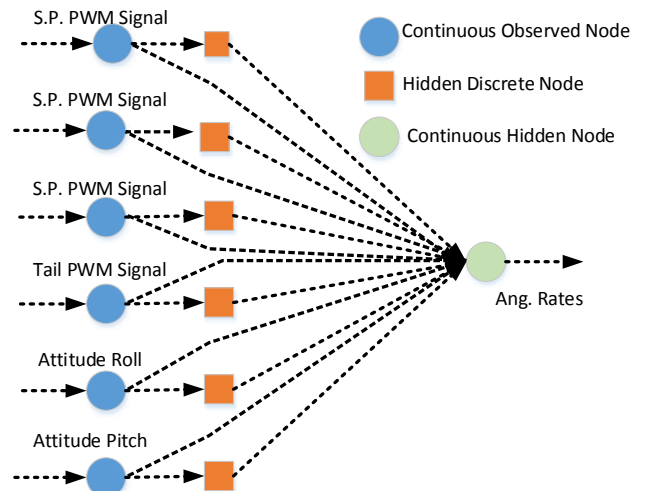

Figure 5. Topology of BME used in proposed telemetry

node) receives information from experts and linearly combines the estimated output value. MATLAB implementation of BME in 'Bayes Net Toolbox' [28] has been used in your system. Fig. 5 shows the topology of the BME model used in the proposed system. As the dimensionality of the input data is 6, 6 experts have been used in this application. Increasing the number of states of hidden discrete nodes improves the prediction accuracy for the extreme inputs (which are statistically rare) but at same time increases the computational load exponentially. Thus, with a balanced trade- off between accuracy and computational load, each hidden node has been assumed to have 3 states. For a given input to an expert, hidden node estimates the likelihood values of coefficients corresponding to each state and probability of being in any particular state. The output node probabilistically combines the output of all the experts to give the final output.
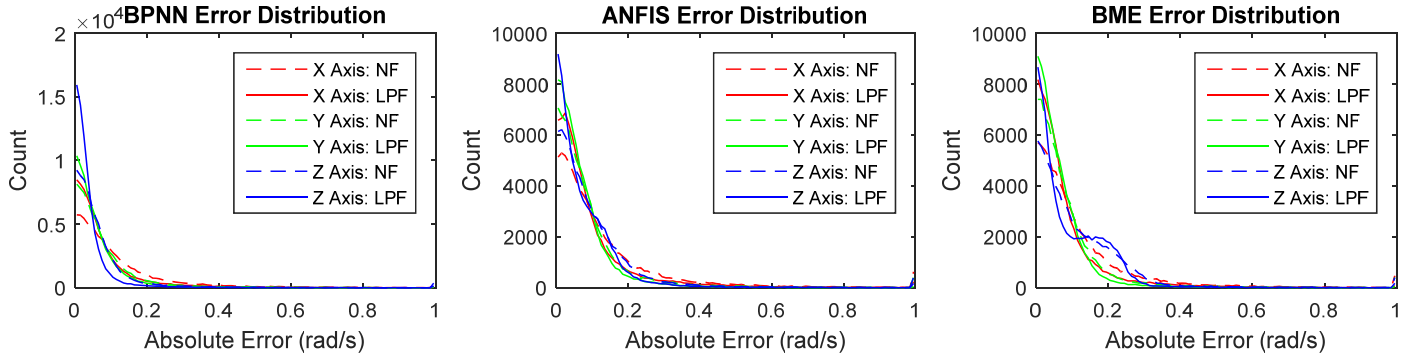
Figure 6. Topology of BME used in proposed telemetry

C. Training ANFIS networks

Adaptive Neuro Fuzzy Inference System (ANFIS) is a type of feed forward network capable of supervised learning [29]. In practice, ANFIS networks are same as Fuzzy Inference Systems (FIS) [30]. Although both BPNN and ANFIS networks have similar topology, there is a significant difference between the two: BPNN optimizes network weights for given activation functions to minimize prediction errors, whereas ANFIS optimizes the shape of fuzzy Membership Functions (MFs) to minimize the prediction errors.

ANFIS networks used in the presented telemetry system use a hybrid learning algorithm [29] (a combination of the least-squares and back-propagation gradient-descent methods) to optimize a Sugeno-type FIS. MATLAB implementation of

ANFIS training in 'Fuzzy Logic Toolbox' [31] has been used in the presented system, which cross-validates the models on a checking dataset to prevent overfitting of the training dataset. During the training phase, different types of MF shapes were tried on Sugeno-type FIS structure to be optimized, but 'Gaussian Bell' shaped input MFs and 'Linear' shaped output MFs delivered acceptable prediction results. Because of memory constraints, the number of MFs was limited to 2 although increasing the number could further improve the prediction accuracy. Three networks for each of the three network types (BPNN, ANFIS and BME) were trained separately to estimate the pilot input component in helicopter dynamics. Fig. 6 shows histogram plots of absolute prediction error of the three networks while predicting the Cartesian components of helicopter's angular rates. To estimate these error
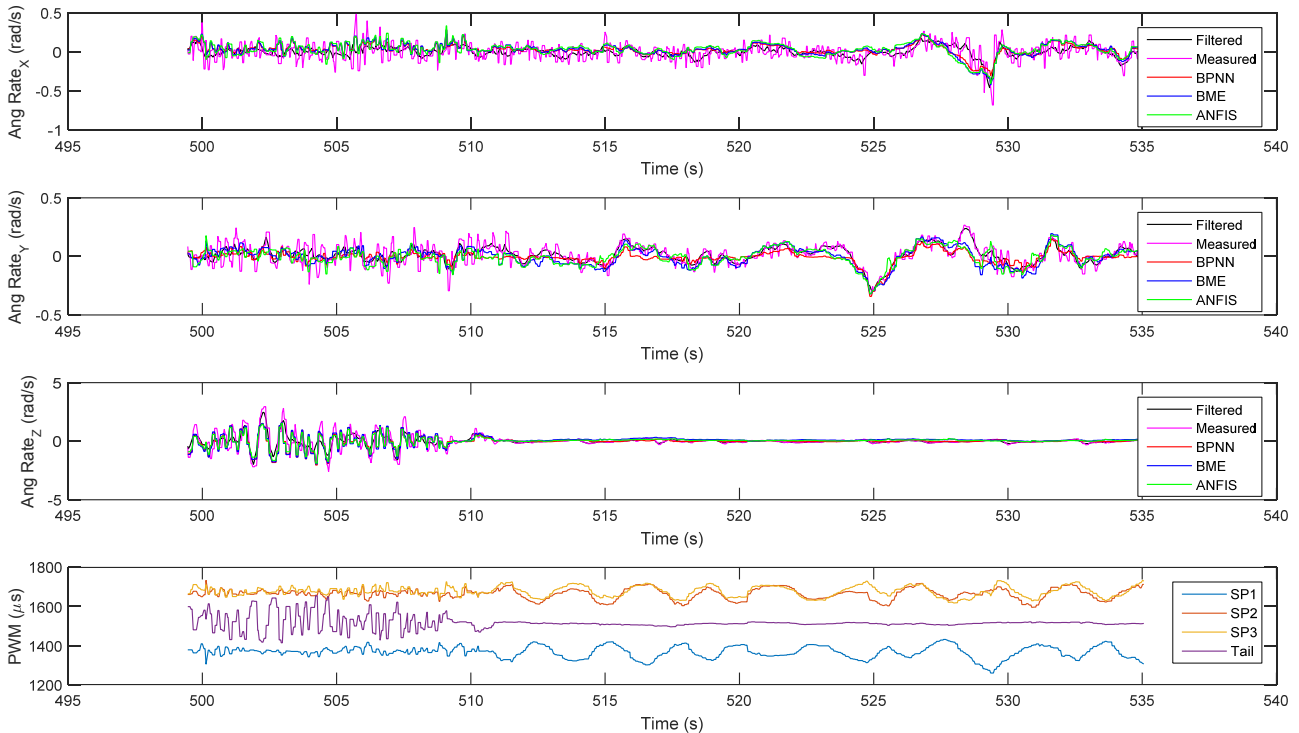

Figure 7. Prediction results from all the three machine learning algotithms

distribution histograms unseen test data from remaining indoor flights (~70K samples). The figure shows 6 histogram plots from the three networks. The plot legend extension 'NF' stands for 'No Filter', and represents prediction error with respect the original data. The plot legend extension 'LPF' stands for 'Low Pass Filter', and represents prediction error with respect the low pass filtered data. The histogram plots in Fig. 6 suggest that the BPNN outperforms both BME and ANFIS by delivering the best fit on training data. BME along with ANFIS, perform similar in modelling the pilot input components in helicopter's noise.

Fig. 7 shows prediction results of angular rates (in rad/s) from all the three types of networks on a short section (35 s) of a test flight. While assessing the prediction results (Fig.7), it was observed that both BME and ANFIS did not perform well on extreme pilot inputs like BPNN, but its performance exceed BPNN on medium/small pilot inputs and hence provided better generalization. This observation arises from two facts that BPNN uses error minimization during its parameter optimization, and both BME and ANFIS use simpler functions to model the pilot input data. This property makes BPNN susceptible to overfitting which is undesirable for ML applications.

V. OUTDOOR TESTING AND RESULTS

To assess the systems performance, the telemetry system was used to estimate the ship air wake patterns generated by YP676 patrol craft in Chesapeake Bay. The pilot was instructed to fly the RC helicopter in a wavy trajectory in potentially active air wake areas. We have hypothesized that the residual angular rates of the helicopter (angular rates after removing the pilot input component and noise) arise from ship air wake, the pattern of these residual angular rates represent the ship air wake patterns. Thus the spatial distribution of the magnitude of the residual angular rates has been presented as the ship air wake intensity pattern.

Fig. 8 shows the residuals for angular rate measurements obtained from BPNN, ANFIS and BME algorithms on the helicopter trajectory. As hypothesized, these prediction residuals represent ship air wake patterns generated by the patrol craft. The trapezium shape in each of the plots represents the flight deck of the YP676. Relative high air wake zones have been marked by freehand contour in the three plots. The intensity map shows the spatial distribution of the magnitude of the residual angular rates. All three ML algorithms were able to detect high air wake zones in the central region of the flight trajectories. Relative sensitivity amongst the three ML algorithms has been assessed on the basis of DC bias (offset) in the intensity plots. It was also observed that the pattern generated using BME, showed least offset in intensity distribution and hence provides more reliable air wake patterns and higher signal to noise ratio. ANFIS networks delivered intermediate accuracy as the air wake patterns generated by it had low DC bias but, also showed sporadic high air wake inconsistent with the air wake patterns generated CFD programs [10]. BPNN performed better on training dataset but showed a significant DC bias in the final air wake patterns. This
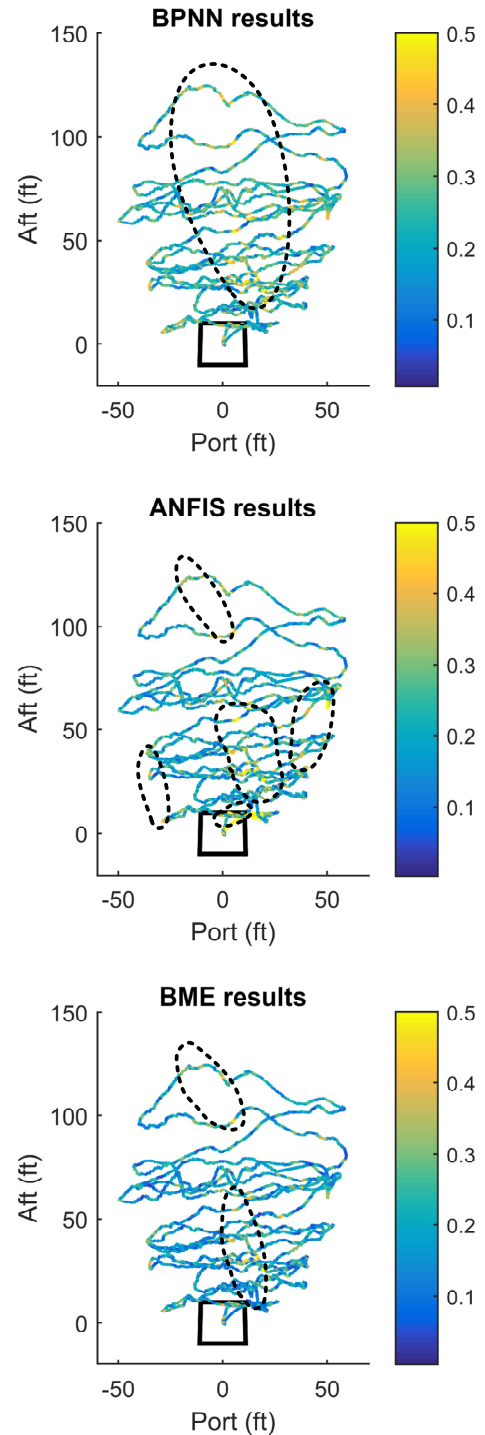

Figure 8. Ship air wake patterns (angular rate residuals in *rad/s*) generated using the three ML algorithms

observation corroborated the fact that BPNN tends to over fit training data and in addition to pilot inputs components, it also modelled high frequency dynamics of the helicopter in angular rates measurements.

VI. CONCLUSION AND FUTURE WORK

The proposed system provides a cost effective solution for mapping of wind turbulence in an area by using off-the-shelf RC helicopters. In general, this wireless telemetry system can be used to estimate any external disturbances on a helicopter. However, this paper demostrated the capability of the system in estimating ship air wake patterns. The telemetry system used three different machine learning algorithms to model the helicopter's response to pilot inputs. This study confirmed that BPNNs are not the best ML tool to model helicoper dynamics as they tend to overfit training data. This study hence helped improve the accuracy of the system over the previous version that utilized only BPNNs [15]. It was concluded that out of the three ML algorithms, BME provided the most reliable air wake patterns. The main point to be noted is that, all these air wake patterns are basically patterns of residual angular rates. To convert these residual angular rates into actual air wakes (in the form of wind velocities), new calibration experiments (involving anemometers) will be required. The authors intend to calibrate this system in the near future in a known, artificially generated wind pattern to map residual angular rates with wind conditions.

ACKNOWLEDGMENTS

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